Fibring (Para)consistent Logics*

Marcelo E. Coniglio[1] Ana Teresa Martins[2]
Amilcar Sernadas[3] Cristina Sernadas[3]

[1] Departamento de Filosofia, IFCH, UNICAMP, Brazil
[2] LIA, Departamento de Computação, UFC, Brazil
[3] CMA, Departamento de Matemática, IST, Portugal

January, 2000

Abstract

The problem of fibring paraconsistent logics is addressed. Such logics raise new problems in the semantics of fibring since previous work assumed verum-functional models. The solution is found in a general notion of interpretation system presentation that "specifies" the intended valuations in some appropriate meta language. Fibring appears as a universal construction in the category of interpretation system presentations, generalizing the results for systems with verum-functional semantics. As an illustration, the fibring of paraconsistent system $\mathcal{C}_1$ and modal system K, while sharing propositional symbols, conjunction, disjunction and implication, is obtained. The fibring of the whole hierarchy $\{\mathcal{C}_n\}_{n\in\mathbb{N}}$ leads to the limit paraconsistent logic $\mathcal{C}_{\mathrm{lim}}$. Fibring is shown to be a promising technique for generating new paraconsistent logics.

1 What is fibring?

In recent years, the problem of combining logics has deserved the attention of many researchers in mathematical logic. Besides leading to very interesting applications whenever it is necessary to work with different logics at the same time, combination of logics is also of interest on purely theoretical grounds [1].

The practical impact of the problem is clear, at least from the point of view of those working in knowledge representation (within artificial intelligence) and in formal specification and verification (within software engineering). Indeed, in these fields, the need for working with several formalisms at the same time is the rule rather than the exception. For instance, in a knowledge representation problem it may be necessary to work with both temporal and deontic aspects. And in a software specification problem it may be necessary to work with both equational and temporal specifications.

*This work started during a visit by the first two authors to CMA at IST. The visit were entirely supported by *Fundação para a Ciência e a Tecnologia*, Portugal.

From a theoretical point of view, one might be tempted, for instance, to look at predicate temporal logic as resulting from the combination of first order logic and propositional temporal logic. But the approach will be significant if and only if general preservation results are available about the mechanism used for combining the logics. For example, if it has been established that completeness is preserved by the combination mechanism $\circ$ and it is known that logic $\mathcal{L}$ is given by $\mathcal{L}' \circ \mathcal{L}''$, then the completeness of the combination $\mathcal{L}$ follows from the completeness of $\mathcal{L}'$ and $\mathcal{L}''$. No wonder that much theoretical effort has been dedicated to establishing preservation results and/or finding preservation counterexamples.

Among the different techniques for combining logics, fibring [6, 7, 15, 16, 17] deserves close study. But what is fibring? The answer can be given in a few paragraphs for the special case of logics with a "propositional base", that is, with propositional variables and connectives of arbitrary arity.

The language of the fibring is obtained by the free use of the language constructors (atomic symbols and connectives) from the given logics. So, for example, when fibring a temporal logic and a deontic logic, "mixed" formulae, like $((G\alpha) \Rightarrow (O(F\beta)))$, appear in the resulting logic. Naturally, in many cases one wants to share some of the symbols. The previous example involves the "constrained" form of fibring imposed by sharing a common propositional part.

At the deductive system level, provided that the two given logics are endowed with deductive systems of the same type, the deductive system of the fibring will be obtained by the free use of the inference rules from both. This approach will be of interest only if the two given deductive systems are "schematic" in the sense that their inference rules are "open" for application to formulae with "foreign" symbols. For instance, when one defines *modus ponens* by the rule MP, $\{(\xi_1 \Rightarrow \xi_2), \xi_1\} \vdash \xi_2$, in some Hilbert system, one may implicitly assume that the instantiation of the "schema variables" ξ_1, ξ_2 by any formulae, possibly with symbols from both logics, is allowed when applying MP in the fibring.

The semantics of fibring is more complicated and, therefore, it is better to consider the special case where both logics have semantics with similar models. Following [15, 17], a possible, quite general, model for many logics with a propositional base is provided by a triple $\langle U, \mathcal{B}, \nu \rangle$ where U is a set (of points, worlds, states, whatever), $\mathcal{B} \subseteq \wp U$, and $\nu(c) : \mathcal{B}^n \to \mathcal{B}$ for each language constructor c of arity $n \geq 0$. In what follows we consider only logics with a semantics given by a class of such triples. Given two such logics $\mathcal{L}', \mathcal{L}''$ what should be the semantics of their fibring? It will be a class of models of the form above, such that at each point $u \in U$ it is possible to extract a model from $\mathcal{L}'$ and one from $\mathcal{L}''$. If symbols are shared, the two extracted models should agree on them. In order to visualize the semantics of fibring, consider the fibring of a propositional linear temporal logic with a propositional linear space logic. Each model of the fibring will be a cloud of points where at each point one knows the time line and the space line crossing there. For instance, at the point $\langle$*Berlin, 10h15m 25 March 2000*$\rangle$ one knows the time line (of past, present and future) of Berlin and the space line (the universe taken as a line for the sake of the example) at that time. However, so far, as sketched above, the semantics of fibring has assumed that the logics at hand are verum-functional and, therefore, does not encompass paraconsistent logics in general.

Paraconsistent logics were introduced in [3] and since then have been the target of continued attention, because of their theoretical and practical significance. For an example of an application see [14, 13] where a paraconsistent modal logic is used for an account of a form of default reasoning. Other systems of paraconsistent modal logic were investigated in [5, 2, 8, 9]. One might wonder if we should try to obtain such mixed logics as the fibring of the underlying modal logic and paraconsistent logic.

The main purpose of this paper is to extend the previous work on the semantics of fibring to a more general notion of systems coping with the case of non verum-functional valuations. In section 2, we introduce the notion of interpretation system presentation as a specification of the intended valuations within a suitable meta logic of equational nature. The valuations themselves appear as the models (algebras) of the specifications. In section 3, we define the notions of (unconstrained and constrained) fibring of such interpretation system presentations. In section 4, we detail the main example of fibring (at the semantic level) the paraconsistent system $\mathcal{C}_1$ and the modal system K. We conclude in section 5 with an assessment of what was achieved and what lays ahead.

2 Specifying valuation semantics

We consider only the case of propositional-based logics. For such logics, the following notion of "signature" is appropriate:

Definition 2.1 A *propositional-based signature* is a family $C = \{C_k\}_{k\in\mathbb{N}}$ where each C_k is a set (of *connectives* of arity k). (The connectives of arity 0 are known as *propositional symbols*.)

We assume given once and for all the set Ξ of *propositional schema variables*, to be used in inference rules, such that $\Xi \cap C_0 = \emptyset$. We denote by $L(C, \Xi)$ the set of *schema formulae* inductively built from C and Ξ.

Later on, we shall need the notion of propositional-based signature morphism from C to C': $h = \{h_k\}_{k\in\mathbb{N}}$ where each h_k is a map from C_k to C'_k. These signatures and morphisms constitute the category **Sig**.

In order to be able to cope with paraconsistent and other non verum-functional logics, it is necessary to deal with valuations that fulfill some requirements. Such requirements are written in a *meta logic* of equational nature that we proceed to define. The environment for the envisaged meta language is a tuple $\langle \Delta, C, \Xi, \phi, v\rangle$ where: Δ is a many-sorted equational signature with a unique sort β (of *truth values*) such that the *verum* symbol $\mathbf{1}$ is in $F_{\epsilon\beta}$; C is a (propositional-based) signature; Ξ is the set of propositional schema variables; ϕ is the sort of schema formulae; v is the valuation symbol.

When dealing with different logics we assume fixed β, ϕ and v, besides Ξ. In other words, when setting up the environment for a specific *object logic* we may choose only the function symbols in Δ (for operations on truth values) and the object propositional-based signature C.

Given such environment we denote by $\Sigma^{\Delta C}$ the many-sorted equational signature obtained by adding v to $F_{\phi\beta}$ on the coproduct of Δ and Σ^C, where Σ^C is the many-sorted equational signature obtained from C as follows: it contains the unique sort ϕ, $F_{\phi^k\phi} = C_k$ for $k > 0$ and $F_{\epsilon\phi} = C_0 \cup \Xi$. Therefore, in $\Sigma^{\Delta C}$ we find two sorts (for truth values and schema formulae), all the propositional connectives and meta variables plus the valuation symbol. We denote by ι^Δ and $\iota^C$the injections within the coproduct of Δ and Σ^C.

We adopt as the *meta language* for specifying the truth values and the valuations the language of the disjunctive, conditional, equational logic for the signature $\Sigma^{\Delta C}$ together with a suitable set X of *β-meta variables* plus a suitable set Y of *ϕ-meta variables.* In this language, an *assertion* is of the general form:

$$(\mathrm{eq}_1 \ \& \ \ldots \ \& \ \mathrm{eq}_n \to \mathrm{eq}'_1 \mid \ldots \mid \mathrm{eq}'_m)$$

with $n \geq 0$ and $m \geq 1$.

A *specification* is a set of such assertions known as *meta axioms.* We are now ready to introduce the notion of semantic structure with the necessary generality to deal with non verum-functional logics.

Definition 2.2 An *interpretation system presentation (isp)* is a tuple $P = \langle C, \Delta, R, S\rangle$ where C is a propositional-based signature, R is the specification of the truth values written within the signature Δ, and S is the specification of the valuations written within the signature $\Sigma^{\Delta C}$.

The two following examples show that this notion is general enough to encompass a wide class of propositional-based logics, from paraconsistent to modal logics.

Example 2.3 *Paraconsistent system $\mathcal{C}_1$* [3]:

- Object signature - C:
    - $C_0 = \{p_n : n \in \mathbb{N}\}$;
    - $C_1 = \{\neg\}$;
    - $C_2 = \{\wedge, \vee, \supset\}$.
- Truth values meta signature - Δ:
    - set of sorts: $\{\beta\}$;
    - $F_{\epsilon\beta} = \{\mathbf{0}, \mathbf{1}\}$;
    - $F_{\beta\beta} = \{-\}$;
    - $F_{\beta\beta\beta} = \{\sqcap, \sqcup, \Rightarrow\}$.

  Recall that β and $\mathbf{1}$ are mandatory.
- Truth values meta axioms - R: an adequate set of axioms for Boolean algebra, inter alia

- $(\; \to x_1 \sqcap x_1 = x_1)$.

- Valuations meta axioms - S:
  - $(\; \to v(y_1 \wedge y_2) = \sqcap(v(y_1), v(y_2)))$;
  - $(\; \to v(y_1 \vee y_2) = \sqcup(v(y_1), v(y_2)))$;
  - $(\; \to v(y_1 \supset y_2) = \Rightarrow(v(y_1), v(y_2)))$;
  - $(v(y) = \mathbf{0} \to v(\neg\, y) = \mathbf{1})$;
  - $(v(\neg\,\neg\, y) = \mathbf{1} \to v(y) = \mathbf{1})$;
  - $(v(y_2^\circ) = \mathbf{1} \;\&\; v(y_1 \supset y_2) = \mathbf{1} \;\&\; v(y_1 \supset \neg\, y_2) = \mathbf{1} \to v(y_1) = \mathbf{0})$;
  - $(v(y_1^\circ) = \mathbf{1} \;\&\; v(y_2^\circ) = \mathbf{1} \to v((y_1 \wedge y_2)^\circ) = \mathbf{1})$;
  - $(v(y_1^\circ) = \mathbf{1} \;\&\; v(y_2^\circ) = \mathbf{1} \to v((y_1 \vee y_2)^\circ) = \mathbf{1})$;
  - $(v(y_1^\circ) = \mathbf{1} \;\&\; v(y_2^\circ) = \mathbf{1} \to v((y_1 \supset y_2)^\circ) = \mathbf{1})$.

  As usual, y° is an abbreviation of $\neg(y \wedge \neg\, y)$.

Example 2.4 *Modal system K* [10]:

- Object signature - C:
  - $C_0 = \{p_n : n \in \mathbb{N}\} \cup \{t\}$;
  - $C_1 = \{\neg, L\}$;
  - $C_2 = \{\wedge, \vee, \supset\}$.
- Truth values meta signature - Δ:
  - set of sorts: $\{\beta\}$;
  - $F_{\epsilon\beta} = \{\mathbf{0}, \mathbf{1}\}$;
  - $F_{\beta\beta} = \{-, \Box\}$;
  - $F_{\beta\beta\beta} = \{\sqcap, \sqcup, \Rightarrow\}$.
- Truth values meta axioms - R: axioms for Boolean algebra, plus
  - $(\; \to \Box(\mathbf{1}) = \mathbf{1})$;
  - $(\; \to \Box(\sqcap(x_1, x_2)) = \sqcap(\Box(x_1), \Box(x_2)))$.
- Valuations meta axioms - S:
  - $(\; \to v(t) = \mathbf{1})$;
  - $(\; \to v(\neg\, y) = -(v(y)))$;
  - $(\; \to v(Ly) = \Box(v(y)))$;
  - $(\; \to v(y_1 \wedge y_2) = \sqcap(v(y_1), v(y_2)))$;
  - $(\; \to v(y_1 \vee y_2) = \sqcup(v(y_1), v(y_2)))$;
  - $(\; \to v(y_1 \supset y_2) = \Rightarrow(v(y_1), v(y_2)))$.

Given an interpretation system presentation $P = \langle C, \Delta, R, S \rangle$, the interpretation structures themselves appear as models of the presentation within the adopted meta logic. We denote by $M(P)$ the class of such models:

$$M(P) = \{ \mathcal{A} \in \mathbf{Mod}(\Sigma^{\Delta C}, \iota^{\Delta}(R) \cup \iota^{C}(S)) : \ \mathcal{A}|_{\iota^C} = \mathrm{TA}(\Sigma^C) \}.$$

That is, a model is an algebra over the equational signature $\Sigma^{\Delta C}$ satisfying the specifications (for truth values and valuations) such that the reduct to the equational signature for the schema formulae is exactly the (free) term algebra $\mathrm{TA}(\Sigma^C)$ over that signature. Therefore, $\mathcal{A}_\phi = L(C, \Xi)$ and the (object) connectives are not interpreted. For instance, $\wedge_{\mathcal{A}}(y_1, y_2) = y_1 \wedge y_2$.

Definition 2.5 Given a model $\mathcal{A} \in M(P)$ and a schema formula γ, we say that $\mathcal{A} \Vdash \gamma$ ($\mathcal{A}$ *satisfies* γ) iff $v_{\mathcal{A}}(\gamma) = \mathbf{1}_{\mathcal{A}}$, that is, the valuation of γ is the verum value.

It is now obvious how to define entailment, validity and the other usual semantic notions. Returning to the first example (paraconsistent system $\mathcal{C}_1$), it is straightforward to verify that the models in $M(P)$, up to isomorphism because of the built-in freedom in the truth values, are in bijection with the paraconsistent valuations introduced in [4]. With respect to the second example (modal system K), it is also straightforward to verify that every Kripke model has a counterpart in $M(P)$: consider the algebra of truth values given by the powerset of the set of worlds. Furthermore, every general model in [17] also has a counterpart in $M(P)$: take $\langle \mathcal{B}, \nu \rangle$ as the algebra of the truth values. In both cases, the extra models do not change the entailment.

Note that it is easy to extend the first example in order to set up the isp's for the whole hierarchy $\mathcal{C}_n$ by specifying the paraconsistent n-valuations introduced in [11].

Before turning our attention to the problem of fibring isp's, we establish the category $\mathbf{Isp}(\Delta)$ of isp's with the same signature Δ for truth values by using the following notion of isp morphism:

$$h : P \to P'$$

iff h is an object signature morphism from C to C' such that:

- for each $r \in R$, $\hat{h}(r)$ is entailed from R' in the meta logic;
- for each $s \in S$, $\hat{h}(s)$ is entailed from S' in the meta logic;

where $\hat{h}$ is the unique map from the meta language of assertions over $\Sigma^{\Delta C}$ to the meta language of assertions over $\Sigma^{\Delta C'}$ that extends the signature morphism h. In the sequel, we also use the forgetful functor N from $\mathbf{Isp}(\Delta)$ to $\mathbf{Sig}$.

3 Fibring non truth-functional logics

We assume that we are given two isp's P' and P'' sharing the same signature Δ for the truth values. This assumption makes the notion of fibring much simpler while retaining the key problems related to dealing with non verum-functional logics.

Following the method proposed in [15], we start by considering the notion of *unconstrained fibring* that corresponds to combining the two isp's without sharing any of the symbols of the object signatures C' and C''. That is, if we so combine $\mathcal{C}_1$ and K we shall get in the result of the fibring two different symbols for conjunction, disjunction, etc. This construction appears as the coproduct of P' and P'' in the category $\mathbf{Isp}(\Delta)$. Therefore,

$$P' \oplus P'' = \langle C, \Delta, R, S \rangle$$

where:

- $C = C' \oplus C''$ within **Sig** with the injections i' and i'';
- $R = \hat{i}'(R') \cup \hat{i}''(R'')$;
- $S = \hat{i}'(S') \cup \hat{i}''(S'')$.

For those familiar with the hierarchy $\mathcal{C}_n$, it should be obvious that the unconstrained fibring of $\mathcal{C}_m$ and $\mathcal{C}_n$ collapses into $\mathcal{C}_{\max\{m,n\}}$. Furthermore, the unconstrained fibring of whole hierarchy $\{\mathcal{C}_n\}_{n \in \mathbb{N}}$ is the limit paraconsistent logic $\mathcal{C}_{\text{lim}}$ as defined in [12]. (Although we presented the notion of fibring of only two isp's, it is straightforward to do it for an arbitrary (small) family of isp's.)

Again following the method in [15], it is easy to introduce the notion of *constrained fibring by sharing connectives and/or propositional symbols* that corresponds to combining the two isp's while sharing some of the symbols of the object signatures C' and C'': the construction appear as a co-Cartesian lifting by the functor N along the signature coequalizer for the envisaged pushout of the signatures. We refrain from dwelling further on this construction since it does not bring any insight to the main issue of this paper: the fibring of logics possibly with non verum-functional semantics.

For an interesting example of constrained fibring see the next section where we present the unconstrained fibring of $\mathcal{C}_1$ and K while sharing the propositional symbols, conjunction, disjunction and implication.

4 Application: Modal paraconsistent logic

The idea is to combine $\mathcal{C}_1$ and K by fibring them while sharing the propositional symbols, conjunction, disjunction and implication. More precisely, given the isp $P' = \langle C', \Delta', R', S' \rangle$ for $\mathcal{C}_1$ (in the first example of section 2) and $P'' = \langle C'', \Delta'', R'', S'' \rangle$ for K (in the second example of that section), we want the isp obtained by fibring P' and P'' while sharing the symbols mentioned above.

Clearly, we immediately face a problem: $\Delta' \neq \Delta''$ but, according to the previous section, we only know how to build a fibring if the two given isp's have the same truth values equational signature. Fortunately, $\Delta' \subset \Delta''$. So, we are tempted to adopt the latter for both isp's. Of course, we should ponder the differences between the original P' and its replacement $Q' = \langle C', \Delta'', R', S' \rangle$ for $\mathcal{C}_1$. No significant differences exist: the extra symbol $\Box$ and its semantics does not affect the entailment.

Therefore, we proceed by fibring Q' and P'' both with the truth values signature Δ''. The problem now is how to impose the sharing of the propositional symbols, conjunction, disjunction and implication for the envisaged constrained fibring.

We work first in the category **Sig**. Consider the propositional-based signature B as follows:

- $B_0 = \{p_n : n \in \mathbb{N}\}$;
- $B_2 = \{\wedge, \vee, \supset\}$;
- $B_k = \emptyset$ for the other values of k.

The propositional-based signature $C' \overset{f'Bf''}{\oplus} C''$ of the envisaged constrained fibring is obtained by the pushout of $C' \overset{f'}{\leftarrow} B \overset{f''}{\rightarrow} C''$ where f' and f'' are the obvious inclusions.

The unique morphism z from the coproduct $C' \oplus C''$ to $C' \overset{f'Bf''}{\oplus} C''$ is now used to obtain the envisage isp as briefly described at the end of the previous section. The isp we want,

$$Q' \overset{f'Bf''}{\oplus} P'',$$

corresponding to the constrained sharing of Q' and P'' while sharing the symbols in B, appears as the co-Cartesian lifting by the functor N along z on the isp $Q' \oplus P''$. That is,

$$Q' \overset{f'Bf''}{\oplus} P'' = \langle C' \overset{f'Bf''}{\oplus} C'', \Delta'', z(i'(R') \cap i''(R'')), z(i'(S') \cap i''(S'')) \rangle$$

Note that we find in the result two negations: $z(i'(\neg))$ coming from C' and $z(i''(\neg))$ coming from C''. The former is a paraconsistent negation and the latter is the classical negation inherited from K. Clearly, the (classical) strong negation "tilde" inherited from $\mathcal{C}_1$ collapses into $z(i''(\neg))$.

The models of the resulting isp are algebras over

$$\Sigma^{\Delta''(C' \overset{f'Bf''}{\oplus} C'')}$$

satisfying the specifications inherited from the two given isp's. They are modal algebras enriched with a paraconsistent operation (for negation).

5 Outlook

The main result of this paper is a general semantics for fibring propositional-based logics encompassing systems with non verum-functional valuations. This goal was achieved by recognizing that such valuations are specified in some appropriate meta logic. Although restricted to systems with a finitary propositional base, the proposed semantics deals with a wide variety of logics from paraconsistent to modal and intuitionistic systems.

Within this setting, fibring appears as a universal construction within the underlying category, generalizing previous results for verum-functional systems [15]. The main open problem is completeness preservation: in which conditions will fibring of logic systems (endowed with semantic presentations as proposed) preserve completeness? Note that a quite general positive answer to this question was given in [17] for the case of systems with verum-functional semantics.

Other lines of research are obvious towards relaxing the assumptions of this paper. For instance, we may want to work with more general object logics (e.g., what about predicate logics?), or with a more general meta logic, or with a more general universe of truth values (e.g., allowing several distinguished values).

Still other lines of research are related to more general forms of fibring, namely "heterogeneous" forms of fibring where we want to combine two (or more) systems that are defined in quite different forms (either at the deductive system level or at the semantics level).

In the front of paraconsistent logic, it will be most interesting to relate the paraconsistent modal logics obtained as fibrings with those already analyzed in the literature and to explore the proposed mechanism for generating new paraconsistent logics.

Acknowledgments

The authors are grateful to Walter Carnielli and Carlos Caleiro for many helpful discussions on relevant topics. This work was partially supported by *Fundação para a Ciência e a Tecnologia* (Portugal), the PRAXIS XXI Projects P/MAT/10002/1998 ProbLog and 2/2.1/TIT/1658/95 LogComp, the ESPRIT IV Working Groups 22704 ASPIRE and 23531 FIREworks, as well as by *CNPq* (Brazil) and the PROBRAL Project on combining deductive systems financed by *CAPES* (Brazil).

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