Possible-Translations Semantics

(Extended Abstract)

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Abstract

This text aims at providing a bird's eye view of possible-translations semantics ([10, 24]), defined, developed and illustrated as a very comprehensive formalism for obtaining or for representing semantics for all sorts of logics. With that tool, a wide class of complex logics will very naturally turn out to be (de)composable by way of some suitable combination of simpler logics. Several examples will be mentioned, and some related special cases of possible-translations semantics, among which are society semantics and non-deterministic semantics, will also be surveyed.

1 Logics, translations, possible-translations

Let a *logic* $\mathcal{L}$ be a structure of the form $\langle \mathcal{S}, \Vdash \rangle$, where $\mathcal{S}$ denotes its *language* (its set of *formulas*) and $\Vdash \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$ represents its associated *consequence relation* (cr), somehow defined so as to embed some formal model of reasoning. Call any subset of $\mathcal{S}$ a *theory*. As usual, capital Greek letters will denote theories, and lowercase Greek will denote formulas; a sequence such as $\Gamma, \alpha, \Gamma' \Vdash \Delta', \beta, \Delta$ should be read as asserting that $\Gamma \cup \{\alpha\} \cup \Gamma' \Vdash \Delta' \cup \{\beta\} \cup \Delta$.

Morphisms between any two of the above structures will be called *translations*. So, given any two logics, $\mathcal{L}_1 = \langle \mathcal{S}_1, \Vdash_1 \rangle$ and $\mathcal{L}_2 = \langle \mathcal{S}_2, \Vdash_2 \rangle$, a mapping $t : \mathcal{S}_1 \to \mathcal{S}_2$ will constitute a translation from $\mathcal{L}_1$ into $\mathcal{L}_2$ just in case the following holds:

(T1) $\Gamma \Vdash_1 \Delta \Rightarrow t(\Gamma) \Vdash_2 t(\Delta)$

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A translation is said to be *conservative* in case the converse of (T1) holds, i.e.:

(T2) $\Gamma \Vdash_1 \Delta \Leftarrow t(\Gamma) \Vdash_2 t(\Delta)$

Given a logic $\mathcal{L}=\langle \mathcal{S}, \Vdash \rangle$, a *possible-translations representation* (ptr) over it is a structure of the form $\langle \mathsf{Log}, \mathsf{Tr}, \mathsf{Reg} \rangle$, where $\mathsf{Log} = \{\langle \mathcal{S}_j, \Vdash_j \rangle\}_{j \in J}$ is an indexed set of logics (also called *factors* or *ingredients* of this ptr), $\mathsf{Tr} = \{t_j : \mathcal{S} \to \mathcal{S}_j\}_{j \in J}$ is an indexed set of translations, and $\mathsf{Reg} \subseteq \mathsf{Pow}(\mathsf{Tr})$. To any such ptr one can immediately associate three levels of consequence relations: A *local* $\mathsf{pt\text{-}cr}$, $\Vdash^j_{\mathsf{pt}} \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$, for each $t_j \in \mathsf{Tr}$, a *regional* $\mathsf{pt\text{-}cr}$, $\Vdash^R_{\mathsf{pt}} \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$, for each $R \in \mathsf{Reg}$, and a *global* $\mathsf{pt\text{-}cr}$, $\Vdash_{\mathsf{pt}} \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$. These relations will be defined by setting:

(L-pt) $\Gamma \Vdash^j_{\mathsf{pt}} \Delta$ iff $t_j(\Gamma) \Vdash_j t_j(\Delta)$

(R-pt) $\Gamma \Vdash^R_{\mathsf{pt}} \Delta$ iff $(\backepsilon t_j \in R)[\Gamma \Vdash^j_{\mathsf{pt}} \Delta]$,
where $\backepsilon$ is some (generalized) quantifier

(G-pt) $\Gamma \Vdash_{\mathsf{pt}} \Delta$ iff $(\forall R \in \mathsf{Reg})[\Gamma \Vdash^R_{\mathsf{pt}} \Delta]$

Obviously, (L-pt) is just a particular case of (R-pt). Taking $\mathsf{Reg} = \{\{t_j\} : t_j \in \mathsf{Tr}\}$ makes the regional $\mathsf{pt\text{-}cr}$ perfectly dispensable —we will call any ptr with that characteristics a *simple* ptr and write it more simply as $\langle \mathsf{Log}, \mathsf{Tr} \rangle$. There are usually many ways of obtaining the same global $\mathsf{pt\text{-}cr}$. Suppose for instance that '$\backepsilon = \forall$' in (R-pt). Then, $\Vdash_{\mathsf{pt}}$ will be exactly the same, for every Reg such that $\bigcup \mathsf{Reg} \supseteq \mathsf{Tr}$.

Given two logics $\mathcal{L}_1=\langle \mathcal{S}_1, \Vdash_1 \rangle$ and $\mathcal{L}_2=\langle \mathcal{S}_2, \Vdash_2 \rangle$, we will say that $\mathcal{L}_1$ is *sound* with respect to $\mathcal{L}_2$ in case $\Vdash_1 \subseteq \Vdash_2$. Similarly, we will say that $\mathcal{L}_1$ is *complete* with respect to $\mathcal{L}_2$ in case $\Vdash_1 \supseteq \Vdash_2$. Notice that translations can be endomorphisms. In particular, any logic is sound and complete with respect to itself, the identity endomorphism always constituting thus a trifling example of a ptr. A ptr over a logic $\mathcal{L}=\langle \mathcal{S}, \Vdash \rangle$ is said to be *adequate* in case $\mathcal{L}$ is sound and complete with respect to $\langle \mathcal{S}, \Vdash_{\mathsf{pt}} \rangle$. Thus, an adequate ptr can be seen as a way of combining a set of translations so as to obtain a very particular conservative translation. Finally, a *possible-translations semantics* (pts) is simply a possible-translations representation in which all factors are defined by 'semantic means' (in contrast to, say, 'abstract deductive' or 'proof-theoretical' means). This characterization certainly looks very vague, but I will show in more detail in the following subsections how the canonical semantic notions work and how they can be seen as special cases of simple pts, according to the above definitions.

One last methodological discrimination is sometimes useful. In case one starts with a logic $\mathcal{L}$ and then finds a set of factors for it in an adequate ptr, one will call the process *splitting logics*; in case one starts with the factors and then build a logic for which the corresponding ptr is adequate, the process will be called *splicing logics*. The immense majority of examples from the literature on *combining logics* is of a more synthetic character: More and more logics are spliced as time goes by. Here, on the contrary, it will be often natural to use ptr's in order to analyze some given logics, splitting them into simpler components in order to understand them. *Frango ut patefaciam.*

Digression 1.1 (*Categorial*) If one considers the category where logics are the objects and translations are the arrows, the diagrams we get for the ptr's all look like there were sunbeams irradiating from a common core. The logic that originates from the combination can be seen as the colimit of this diagram. In [11] the authors show how to generalize this construction for arbitrary diagrams. This should be compared to what is done in [30] in understanding *fibring* (a more general form of combination, check [23, 4]) as a categorial construction. A first advance in that direction, generalizing the basic construction of fibring, can be found in [16]. A different semantically driven generalization of fibring, *cryptofibring*, is categorially investigated in [7]. □

Digression 1.2 (*Historical*) Possible-translations semantics were first introduced in [9], restricted to the use of finite-valued factors. The embryo was then frozen for a period, and in between 1997 and 1998 it was publicized under the denomination 'non-deterministic semantics', in [12], and in several talks by Carnielli and a few by myself. Noticing that the non-deterministic element was but a particular accessory of the more general picture, from 1999 on the semantics retook its earlier denomination ([10, 24, 14, 15, 26]). □

1.1 What is a logic?

This is a question that will *not* be answered in this section. Any number of answers to it can be found in the literature, if you dig hard enough. I will here instead show how some among the most popular answers can be recast in the present framework.

Given a logic $\mathcal{L} = \langle \mathcal{S}, \Vdash \rangle$ as above, we will call it *scottian* in case its cr is subject to the following restrictions (cf. [28]):

(C1)	$(\Gamma, \varphi \Vdash \varphi, \Delta)$	(overlap)
(C2)	$(\Gamma \Vdash \varphi, \Delta)$ and $(\Gamma', \varphi \Vdash \Delta') \Rightarrow (\Gamma', \Gamma \Vdash \Delta, \Delta')$	(cut)
(C3)	$(\Gamma \Vdash \Delta) \Rightarrow (\Gamma', \Gamma \Vdash \Delta, \Delta')$	(dilution)

Call any clause of the form $\Gamma \Vdash \Delta$ an *inference*. Theories that appear at the left-hand side of the $\Vdash$ are also dubbed *countertheories*, or *premises* assumed by the inference; theories that appear at the right-hand side of the $\Vdash$ are also called *alternatives* sanctioned by the inference. A *tarskian* cr (cf. [32]) is a particular case of a scottian cr, in which each inference has a single formula as alternative (no real 'alternative' in that case, is it?). Such alternative is often called *consequence* of the inference. Tarskian logics are also called *single-conclusion*, in contrast to the more symmetrical (*multiple-premise*) *multiple-conclusion* scottian logics. It would be just as natural, of course, to consider here a *countertarskian* logic to be defined by the same restrictions above, but on a single-premise-multiple-conclusion environment. Very uncommon in practice, the countertarskian case works pretty much like the tarskian case in most circumstances. Below I will only mention countertarskian logics explicitly, thus, when relevant.

Here are some degenerate examples of logics. Let a logic $\langle \mathcal{S}, \Vdash \rangle$ be called *overcomplete* in case its cr is characterized by one of the following universal properties:

(C0.0.0) $(\Gamma \Vdash \Delta)$ (triviality)
(C0.0.1) $(\Gamma, \alpha \Vdash \Delta)$ (nihilism)
(C0.1.0) $(\Gamma \Vdash \beta, \Delta)$ (dadaism)
(C0.1.1) $(\Gamma, \alpha \Vdash \beta, \Delta)$ (semitriviality)

Note, by the way, that THE trivial logic is characterized by the nonproper cr over the language $\mathcal{S}$. Clearly, tarskian logics must identify trivial and dadaistic logics, and identify nihilistic and semitrivial logics. When we talk about THE dadaistic logic in a given language we will be referring to the logic having a non-trivial dadaistic cr. Similarly, THE nihilistic logic will refer to the logic having a non-trivial nihilistic cr, and THE semitrivial logic will denote the logic having a non-dadaistic non-nihilistic cr.

A formula β of a logic $\mathcal{L}$ is said to be a *thesis* of this logic in case $(\Gamma \Vdash \beta, \Delta)$, for any choice of Γ and Δ; an *antithesis* of this logic is any formula α such that $(\Gamma, \alpha \Vdash \beta)$, for any choice of Γ and Δ. An arbitrary thesis is sometimes denoted by $\top$, and an arbitrary antithesis is sometimes denoted by $\perp$.

Theorem 1.1.1 (i) Every multiple-conclusion overcomplete logic is scottian. Every single-conclusion overcomplete logic is tarskian.
(ii) The empty language defines a unique scottian / tarskian logic.
(iii) Any arbitrary intersection of scottian / tarkian logics defined over some fixed language defines a scottian / tarskian logic. □

Theorem 1.1.2 Fix some scottian / tarskian logic $\mathcal{L}$ over some non-empty language $\mathcal{S}$. Then:
(i) $\mathcal{L}$ is the trivial logic iff there is at least one formula in its language which is both a thesis and an antithesis of $\mathcal{L}$.
(ii) $\mathcal{L}$ is the nihilistic logic iff all of its formulas are antitheses of it.
(iii) $\mathcal{L}$ is the dadaistic logic iff all of its formulas are theses of it.
(iv) $\mathcal{L}$ is the semitrivial logic iff any formula implies any other (or the same) formula, but no antitheses nor theses are present in the language of this logic. □

Several other restrictions and extensions of the above notion of logic are studied in [25], from an abstract viewpoint. As in that paper, a logic here will be called *minimally decent* in case it is not overcomplete.

1.2 What is the canonical notion of entailment?

Let $\mathcal{V}$ denote an arbitrary set of *truth-values*, where $\mathcal{D}^{\mathcal{V}} \subseteq \mathcal{V}$ denotes its subset of *designated* values (the 'true truth-values'), and $\mathcal{U}^{\mathcal{V}} = \mathcal{V} \setminus \mathcal{D}^{\mathcal{V}}$ denotes its subset of *undesignated* values (the 'false truth-values'). Given a language $\mathcal{S}$, let a *valuation* over it be any mapping $\S^{\mathcal{V}} : \mathcal{S} \to \mathcal{V}$. Call any collection of valuations over $\mathcal{S}$ a (*scottian*) *semantics* sem over $\mathcal{S}$. This semantics will be called κ-*valued* if κ is the greatest cardinality of truth-values of the valuations in sem, that is, $\kappa = \mathsf{sup}_{\S^{\mathcal{V}} \in \mathsf{sem}}(|\mathcal{V}|)$. To any valuation $\S^{\mathcal{V}}$ and any semantics sem one can associate *canonical* notions of *local entailment*, $\vDash^{\S}_{\mathsf{sem}} \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$ and *global entailment*, $\vDash_{\mathsf{sem}} \subseteq \mathsf{Pow}(\mathcal{S}) \times \mathsf{Pow}(\mathcal{S})$, by setting:

(L-ce) $\Gamma \vDash^{\S^{\mathcal{V}}}_{\mathsf{sem}} \Delta$ iff $(\S^{\mathcal{V}}(\Gamma) \cap \mathcal{U}^{\mathcal{V}} \neq \varnothing$ or $\S^{\mathcal{V}}(\Delta) \cap \mathcal{D}^{\mathcal{V}} \neq \varnothing)$

(G-ce) $\Gamma \vDash_{\mathsf{sem}} \Delta$ iff $(\forall \S^{\mathcal{V}} \in \mathsf{sem})[\Gamma \vDash^{\S^{\mathcal{V}}}_{\mathsf{sem}} \Delta]$

An *ordinary* scottian semantics is one in which a fixed cardinal of designated / undesignated values is set throughout all the valuations of the semantics. Obviously, any semantics can be made ordinary by just adding to each valuation a convenient number of truth-values that will not be used. Similarly to above, a *tarskian* (*ordinary*) *κ-valued semantics* will be defined just like a scottian (ordinary) κ-valued semantics, only that all inferences will have exactly one formula at their right-hand sides.

Theorem 1.2.1 (i) Any scottian / tarskian κ-valued semantics induces at least one scottian / tarskian logic by way of one of its associated canonical entailment relations.
(ii) Consider any covering of the valuations of a given scottian / tarskian semantics. Each layer of the covering can now be said to determine a new (universal) 'regional semantics', and the intersection of all the entailments associated to the latter gives you back the global entailment. □

Given the above results, one sees that any semantic structure of the form $\langle \mathcal{S}, \vDash \rangle$ defines a scottian and a tarskian logic, and the logics corresponding to the global entailment relation can be obtained through the intersection of all local (or regional) entailment relations. As before, given a logic $\mathcal{L} = \langle \mathcal{S}, \Vdash \rangle$ and a semantics sem over $\mathcal{S}$, one can now very naturally talk about $\mathcal{L}$ being *locally sound* with respect to some $\S \in \mathsf{sem}$ in case $\Vdash \subseteq \vDash^{\S}_{\mathsf{sem}}$, and being *globally sound* with respect to sem in case $\Vdash \subseteq \vDash_{\mathsf{sem}}$. Similarly for local and global completeness and adequacy. The statement of the following result parallels that of Theorem 1.1.2.

Theorem 1.2.2 Here is how you can obtain adequate ordinary semantics for each sort of overcomplete logic:
(i) For the trivial logic, consider the empty semantics (empty set of truth-values).
(ii) For the nihilistic logic, consider some semantics whose valuations make everything false.
(iii) For the dadaistic logic, consider some semantics whose valuations make everything true.
(iv) For the semitrivial logic, consider some semantics whose valuations either make everything true or make everything false. □

1.3 What can be done with translations between logics?

The general definitions of translation and of conservative translation that you found at the beginning of the present section were studied in detail in [12, 19], and interesting specializations of these notions were proposed in [20]. Typical examples of everyday translations are given by the endomorphisms that define uniform substitutions in a logic whose language is a free algebra (of formulas). One can here also easily check that:

Theorem 1.3.1 (i) A logic can always be conservatively translated into itself. (ii) To check soundness or completeness of a given logic with respect to some scottian / tarskian semantics amounts to checking the identity mapping from the language into itself to be a translation. □

Here are some degenerate examples of translations:

Theorem 1.3.2 For logics (not necessarily scottian nor tarskian) over some fixed language $\mathcal{S}$:
(i) Any logic is translatable into the trivial logic.
(ii) Any single-conclusion logic is translatable into any logic having a thesis. Any single-premise logic is translatable into any logic having an antithesis.
(iii) The trivial single-conclusion logic is conservatively translatable into any logic having a thesis. The trivial single-premise logic is conservatively translatable into any logic having an antithesis. The semitrivial logic is conservatively translatable into any logic having a thesis but no antitheses, or having an antithesis but no theses.
(iv) Given a logic with no (anti)theses at all, NO logic having a(n anti)thesis whatever is translatable into the former.
(v) Any logic having no theses nor antitheses is translatable into the semitrivial logic. □

Problem: For more esoteric non-scottian logics, such as non-monotonic logics and other context-dependent applications it might seem more natural to work with a definition of translation that directly involves the inferences, instead of the formulas. In that case, a translation from $\langle \mathcal{S}_1, \Vdash_1 \rangle$ into $\langle \mathcal{S}_2, \Vdash_2 \rangle$ had better be defined, say, as a mapping $t : \mathsf{Pow}(\mathcal{S}_1) \to \mathsf{Pow}(\mathcal{S}_2)$ instead of $t : \mathcal{S}_1 \to \mathcal{S}_2$, as before. It might be better as well to think of a logic directly as a set of theories, instead of a set of formulas, endowed with a consequence relation. The properties of these sorts of definitions are yet to be investigated in more detail. An advance in that direction was already made in [17], where the authors conceive tarskian logics as two-sorted first-order structures (the sort of 'formulas' and the sort of 'theories'), and talk about 'transfers' as morphisms among those structures (of which translations between tarskian logics, in the above sense, are but particular cases).

1.4 What are possible-translations semantics?

We have defined above the notion of a possible-translations representation (ptr) based on the combination of a collection of factors through local ($\Vdash^{j}_{\mathsf{pt}}$), regional ($\Vdash^{R}_{\mathsf{pt}}$) and global ($\Vdash_{\mathsf{pt}}$) consequence relations (cr). A possible-translations semantics (pts) was then characterized as a ptr based on factors defined by 'semantic means'. Moreover, the above sections have shown a conventional rendering of the received notion of 'semantics', slightly generalized in accordance with the principles of the theory of valuations (cf. [18]) and of abstract multiple-conclusion deductive systems (cf. [33, 31]).

There are several ways of combining logics. In a very pleasant paper, [3], Blackburn and de Rijke survey the reasons one might have for splicing logics, and propose a catalogue of the forms of combination based on the increasing level of involvement of the ingredient logics: They come up with nice pictures for 'refining structures', then 'classification structures', then 'totally fibred structures'. Another taxonomy is proposed in [8, 4, 29], where 'synchronization' and 'parameterization' appear as distinguished special cases of 'fibring'. How would the general picture for the combination through a possible-translations representation look like?

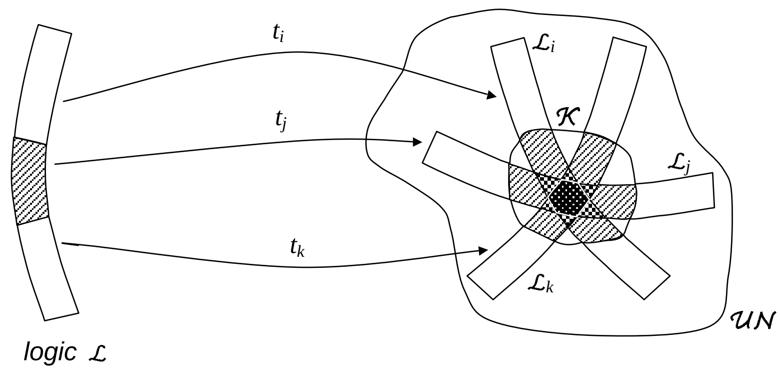

Figure 1: The logical Rosetta Stone.

An insightful analogy may be provided by concentrating on the situation in which a logic is split into its simpler components and comparing it to the deciphering of the 'Rosetta Stone' (cf. [15]). Carved in 196B.C. and found by Napoleon troops in July 1799 near the homonymous village (Rashid) located in the western delta of the Nile, the Rosetta Stone is a basalt slab containing three different inscriptions of a text written by a group of priests to honor the Egyptian pharaoh. Why is it important? Because it finally allowed scholars to decipher the Hyerogliphic writing, a problem that had been open for several hundred years! After the work of Thomas Young, a British physicist, and Jean-François Champollion, a French Egyptologist, the code was finally broken, and a phonetic value was attached to hyeroglyphs that had previously been thought to have a purely symbolic value. How was it done? The three scripts in the stone were the Hyeroglyphic (used for important or religious documents), the Demotic (everyday Egyptian script) and the Greek (language of the rulers of Egypt at that time). With the aid of both Greek and Coptic (language of the Christian descendants of the ancient Egyptians), Champollion was able to decipher the Demotic writing, and from that he was able to trace back the meaning of the Hyerogliphic signs. But how did they know that the three scripts represented the same text, to start with? Because the stone *said so*, at the very end of its Greek inscription! Another beautiful example of self-reference, therefore.

Based on the above story, Figure 1 gives a schematic illustration of what is going on when a ptr is designed. The Rosetta Stone is the 'logical universe' $\mathcal{UN}$ where all ingredient logics can be found, resembling perhaps an egg with the sunny side up. The long curved format of the logic represents the form of reasoning sanctioned by it. You can see that the morphisms (possible-translations) are intended to preserve that format. At a distinguished hachured region of each logic you may find its circumstantial theses and antitheses. Each translation should in particular take theses into theses, and antitheses into antitheses. The region where they can be found in $\mathcal{UN}$ is at its yolk $\mathcal{K}$. The appetizing part is the one in which the ingredients are cooked together so as to give us the corresponding possible-translations structure.

The next result shows some simple examples of ptr and pts:

Theorem 1.4.1 (i) Any logic has an adequate possible-translations representation.
(ii) Any (scottian / tarskian) semantics can be seen as a possible-translations semantics with any positive number of factors. □

One can count now on a more sophisticated interplay between local and global notions at hand: If a scottian / tarskian semantics can be seen as a general way of gluing arbitrary collections of valuations, a possible-translations semantics can be seen as a more general way of gluing collections of any arbitrary kind of previously given semantics.

Call a semantics *unitary* in case it is defined by way of a single valuation, or a single factor; call it *large* in case the cardinality of the set of valuations or the set of factors is at least as big as the cardinality of the underlying language. Obviously, any unitary semantics is ordinary from its very inception; unitary semantics can be made large, and large semantics can always be made ordinary at request, by the addition of redundant valuations or truth-values. We already knew from Theorem 1.2.1(ii) than any scottian / tarskian semantics can be reduced to the intersection of unitary scottian / tarskian semantics; the last result above suggests now that any semantics can ultimately and quite naturally be converted into a large possible-translations semantics whose factors are all unitary semantics themselves.

Moreover:

Theorem 1.4.2 If you are talking about logics characterized by scottian / tarskian entailments, or by simple possible-translations representations:
(i) Global soundness implies local soundness.
(ii) Local completeness implies global completeness.
In overcomplete logics:
(iii) Local soundness automatically transfers to global soundness.
(iii) Global completeness automatically transfers to local completeness. □

1.5 Which logics have adequate semantics?

Right now we have two things called *scottian*: The abstract consequence relations characterized by way of clauses (C1)–(C3) in subsection 1.1 and the semantics to which canonical entailment relations were associated in subsection 1.2. A similar thing can be said about abstract tarskian consequence relations and tarskian semantics. The attentive reader will certainly have noticed, though, that we did not establish a relation between the homonymous creatures! This subsection will correct this slip for the benefit of the interested.

Consider first the tarskian case. Given a single-conclusion logic $\langle \mathcal{S}, \Vdash \rangle$ and a counter-theory $\Pi \subseteq \mathcal{S}$, the *right-closure* of Π, denoted by $\Pi^{\mathbf{c}}$, is the set of all of its derived consequences, that is, the set $\{\pi : \Pi \Vdash \pi\}$.

Theorem 1.5.1 (i) In any tarskian logic, $\Pi^{\mathbf{cc}} = \Pi^{\mathbf{c}}$, that is, $\Pi^{\mathbf{c}} \Vdash \pi \Leftrightarrow \Pi \Vdash \pi$. (ii) In any tarskian logic $\langle \mathcal{S}, \Vdash \rangle$, given arbitrary $\Sigma \cup \Gamma \cup \{\varphi\} \subseteq \mathcal{S}$, to check whether $\Sigma, \Delta \Vdash \varphi$ holds is equivalent to checking whether $(\forall \gamma \in \Gamma)\Sigma \Vdash \gamma$ implies $\Sigma \Vdash \varphi$. □

Theorem 1.5.2 (*Lindenbaum-like*) Each tarskian logic has at least as many (but no less than one) sound tarskian unitary semantics as the number of its right-closed theories. □

Theorem 1.5.3 (*Wójcicki-like*) Every tarskian logic has an adequate semantics. □

Corollary 1.5.4 Every tarskian logic $\langle \mathcal{S}, \Vdash \rangle$ has an adequate ordinary κ-valued semantics, with $\kappa \leq |\mathcal{S}|$. □

The previous result is very general, but a κ-valued semantics is more interesting in case its truth-values are well-behaved with respect to the underlying language, for instance, in case one can count on truth-functionality. The contrast between designated and undesignated values casts though a shadow of *bivalence*. Indeed:

Theorem 1.5.5 (*Suszko-like*) Every tarskian logic has an adequate κ-valued tarskian semantics, for $\kappa \leq 2$. □

Everything can be easily dualized to the counter-tarskian case. Only that now, given a single-premise logic $\langle \mathcal{S}, \Vdash \rangle$ and a theory $\Pi \subseteq \mathcal{S}$, you had better work with the *left-closure* of Π, denoted by ${}^{\mathbf{c}}\Pi$, as the set of all of its deriving premises, that is, the set $\{\pi : \pi \Vdash \Pi\}$. The rest is straightforward to adapt.

I will now briefly show how the above constructions can be modified for the scottian case (cf. [31]). As usual, call $\langle \Sigma, \Pi \rangle$ a *partition* of the set $\Theta \subseteq \mathcal{S}$ in case $\Sigma \cup \Pi = \Theta$ and $\Sigma \cap \Pi = \varnothing$.

Theorem 1.5.6 (*Cut for sets*) Given a scottian logic $\langle \mathcal{S}, \Vdash \rangle$:
If $\Gamma, \Sigma \Vdash \Pi, \Delta$, for every partition $\langle \Sigma, \Pi \rangle$ of Θ then $\Gamma \Vdash \Delta$. □

Theorem 1.5.7 (*L-theorem*) Each scottian logic has some sound scottian unitary semantics. □

Theorem 1.5.8 (*W-theorem*) Any scottian logic has an adequate semantics. □

Corollary 1.5.9 Every scottian logic $\langle \mathcal{S}, \Vdash \rangle$ has an adequate ordinary κ-valued semantics, with $\kappa \leq |\mathcal{S}|$. □

Theorem 1.5.10 (*S-theorem*) Every scottian logic has an adequate κ-valued scottian semantics, for $\kappa \leq 2$. □

One can conclude from the above results that:

Theorem 1.5.11 (i) Every tarskian / scottian logic has an adequate possible-translations semantics, in fact even a possible-translations semantics based on 2-valued factors (copies of classical logic).
(ii) The local and the global consequence relations associated to any simple possible-translations representation or possible-translations semantics based on tarskian / scottian factors is tarskian / scottian. □

It is noteworthy that the above results for canonical semantics have pretty much the same flavor of a pts: Each unitary semantics can be seen as determining a translation, and the intersection of all of the appropriate unitary semantics in each case gives you the desired conservative translation.

2 Further illustrations

We have seen, in the previous section, that every scottian / tarskian logic has an adequate scottian / tarskian (2-valued) semantics. Moreover, any logic (scottian, tarskian, or not) has an adequate possible-translations representation (ptr), and if it has an adequate semantics (scottian, tarskian, or not) then it can be given an adequate possible-translations semantics (pts).

What about other less trivial examples of possible-translations semantics, not obtained by plain use of brute force, as above? Indeed, notice that the previous adequacy results were often either uninformative (when a logic was used to represent itself) or non-constructive (when a κ-valued semantics was posited but no recursive method was presented so as to define it). The situation can be improved in a some cases. In the case of sufficiently expressive finite-valued truth-functional logics, for instance, a constructive method can be designed for the specification of a recursive set of clauses that describe the 2-valued semantics announced by Theorem 1.5.5 (cf. [6, 5]).

Moreover, to get even more concrete, one can use a ptr to provide, say, a pts based on a couple of well-behaved and well-known finite-valued truth-functional factors for logics having NO adequate finite-valued scottian / tarskian truth-functional semantics, as done in [10, 24, 14, 26] for several paraconsistent and paracomplete logics. Also, deductive limits for infinite hierarchies of logics can very naturally be spliced, and decidability transferred from the factors to the product, as in [24, 14]. Moreover, truth-functional finite-valued logics can themselves be split in terms of 2-valued logics, that is, fragments of classical logic ([24, 27]), copies of classical logic can be combined into fragments of modal logics, and so on and so forth.

The final version of the paper will display a few representative such examples in detail.

3 Some other related semantic structures

The advantage of possible-translations semantics lies in its generality. It is no overstatement to assert that pretty much anything that one might want to call a semantics can be recast in the present framework. This leads us immediately to the main disadvantage of possible-translations semantics: its generality! Anything that is universally true can easily turn out to be also universally irrelevant. It is very important thus to characterize some interesting subclasses of possible-translations semantics, defined by stricter terms. Clauses restricting the set of translations or the factors involved are often helpful, often inevitable. With that in mind, *society semantics* ([13, 24, 21, 22]), *dyadic semantics* ([6, 5]), and (dynamic and static) *non-deterministic semantics* ([2, 1]) can all be precisely characterized as specialized forms of possible-translations semantics.

This will be done in detail in the final version of the paper.

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